

Slopes and Equations of Lines

Calculus = Study of how quantities vary with respect to each other. (variables)

Examples :

- 1/ (Motion) How position varies with time.
- 2/ How demand for a product varies with unit cost.
- 3/ How air pressure varies with height above sea level

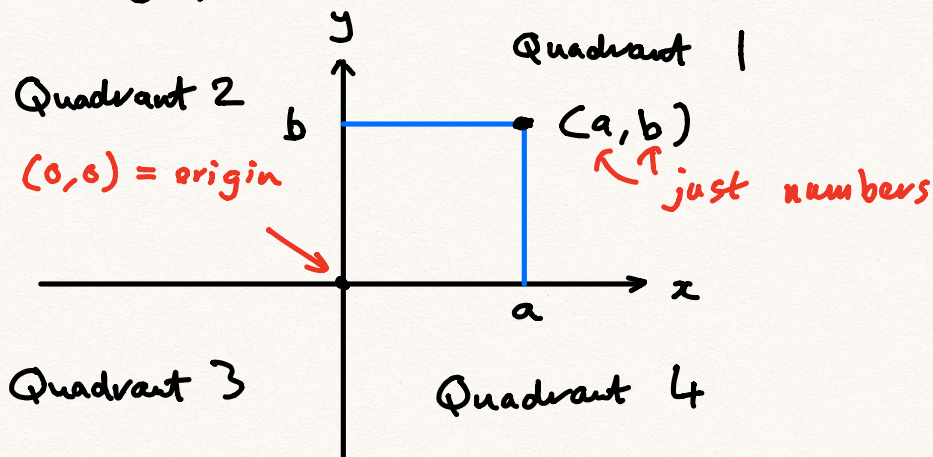
Linear Relationship = Changing one quantity by fixed amount alters the other by fixed amount.

Example You sell a product at a fixed price of \$10. Then revenue (total money earned) varies linearly with respect to the number of units sold. If we sell 10 more units we always earn \$100 more.

Picture :

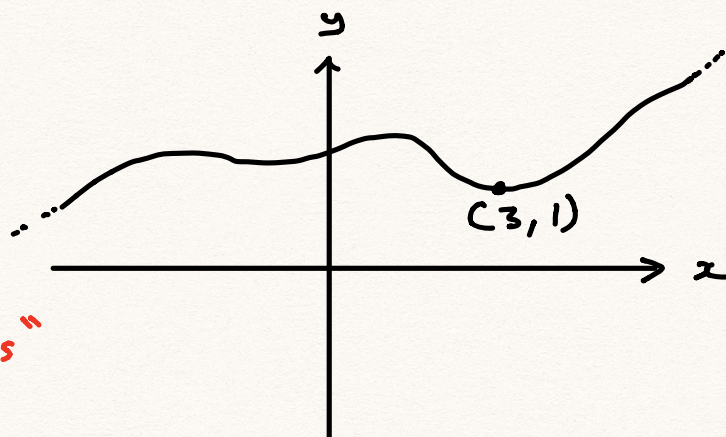


We can represent the relationship between two quantities (variables x and y) using Cartesian coordinates (xy -plane) :



If y varies depending on x this traces out a graph in xy -plane.

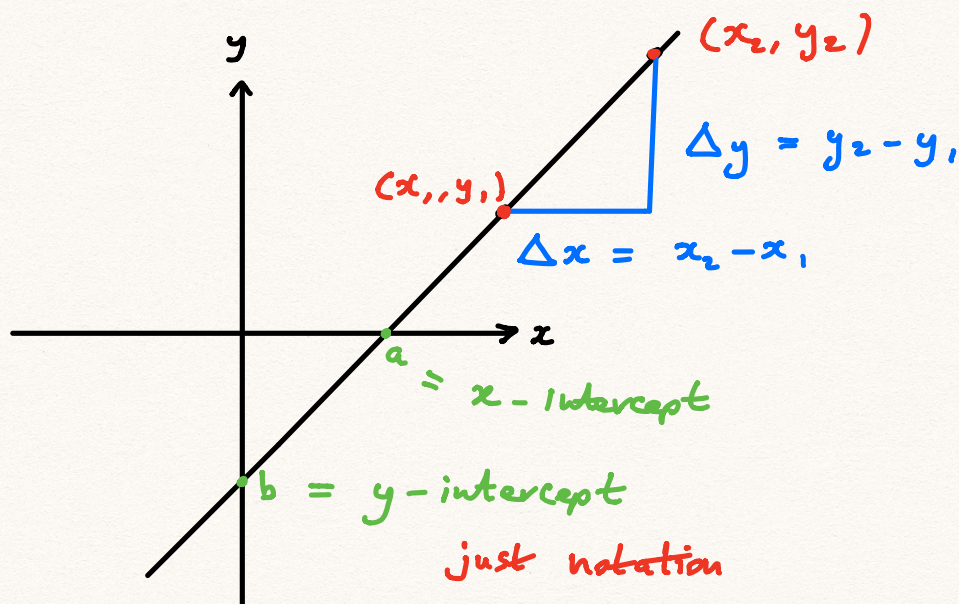
Example :



"implies"
 \Rightarrow When x has value 3, y has value 1.

Relationship between x and y Linear \Rightarrow Graph is straight line

Q/: How can we precisely describe a straight line? Can we use this to describe linear relationships using algebra?

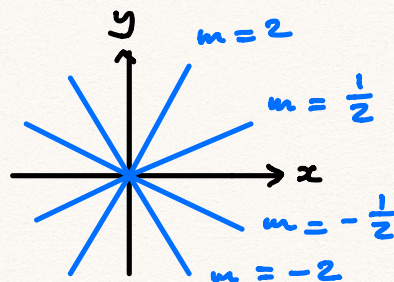


Slope of (non-vertical) straight line $= m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$

Example If line contains $(1, 3)$, $(2, 7)$
 \Rightarrow Slope $= \frac{7-3}{2-1} = \frac{4}{1} = 4$

Remark 1/ Any 2 points on line will give same slope.

2/ Slope = steepness of line



3/ If a line is vertical we say the slope is undefined.

$m = 0 \Rightarrow$ Line horizontal

m undefined \Rightarrow Line vertical

4/ We say 2 lines are parallel if they have same slope. We say two lines are perpendicular if they cross at right-angles.

If two lines are perpendicular \Rightarrow either one is horizontal and other vertical

or

they have slopes m and $-\frac{1}{m}$.

5/ A straight line is completely determined by its slope and a single point on line.

Equations of a (non-vertical) Line :

Slope - Intercept Form : $y = mx + b$
slope \swarrow m \searrow b \swarrow y -intercept

Point - Slope Form : $y - y_1 = m(x - x_1)$
slope \swarrow m
((x_1, y_1) on line)

Equation of vertical line : $x = a$ \swarrow a \swarrow x -intercept

Examples

1/ Find an equation for straight line

containing $(1, 2)$ and $(-1, 6)$.

$$m = \frac{6-2}{-1-1} = \frac{4}{-2} = -2$$

$$\Rightarrow y - 6 = -2(x - (-1))$$

point-slope form

2 Find an equation for straight line perpendicular to $6x + 2y = 1$ which passes through origin.

$$6x + 2y = 1 \Rightarrow y = -3x + \frac{1}{2}$$

$$\Rightarrow \text{slope} = -3$$

$$\Rightarrow \text{perpendicular slope} = \frac{-1}{-3} = \frac{1}{3}$$

$$\Rightarrow \text{perpendicular line is } y = \frac{1}{3}x + 0$$

$$\Rightarrow y = \frac{1}{3}x$$

y-intercept